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NEW ALGORITHM FOR MODELING OF BRONCHIAL TREES

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Abstract. The article presents new conception of 3D model of human bronchial tubes, which represents bronchial tubes extracted from CT images of the chest. The new algorithm which generates new model is an extension of the algorithm (basic algorithm) proposed by Hiroko Kitaoka, Ryuji Takaki and Bela Suki. The basic model has been extended by geometric deformations of branches and noise which occur in bronchial trees extracted from CT images. The article presents comparison of results obtained with the use of the new algorithm and the basic one. Moreover, the discussion of usefulness of generated new models for testing of algorithms for quantitative analysis of bronchial tubes based on CT images is also included.

1 Introduction

The bronchus is an element of respiratory system and lie between trachea and lung bronchioles. The main function of bronchus is to transport air from and into the lungs. Bronchus has a tree structure. The trachea divides into two main bronchi: left and right. The main bronchi are further divided into next branches. Fig. 1 shows the fragment of a bronchial tree.

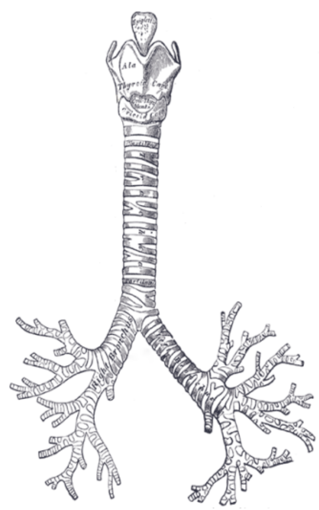


Fig. 1: Front view of cartilages of the larynx, trachea and bronchi [8]

Geometry of the bronchial tree is closely related to the spatial arrangement of the tree. Each branch is assigned to a particular part of a lung volume supplied in air by the branch and its descendants (in the tree hierarchy). This allows to correlate volume of air delivered through a branch with its spatial arrangement in a lung. Therefore, a branch size and direction is determined by size and geometry of the corresponding volume. More details on the rules of lung volume division are presented in the next section.

In recent years several researchers proposed some models of bronchial tree which vary in accuracy. Weibel [19] and Horsfield et al. [9] proposed structural models of airway that include airway dimensions and connectivity. However, these models do not include information about the spatial arrangement of the airway structure, and hence they are limited to modeling lung function in one dimension. Although there are several geometric airway models such as those proposed by Nelson, Manchester [13] and Martonen, Yang et al. [12], these models are still limited to 2D.

The rapid development of three-dimensional technology, which occurred in the 90s of XX century and continues today, and demand from the medical community for 3D models of bronchial tree led to the construction of a number of three-dimensional models of bronchial tree. The first 3D models were constructed in 80s of the XX century - Chen, Shiah, et al. [4]. One of the first 3D models, that is worth mentioning, presented by Kitaoka, Takaki et al. [11] is based on 9^{th} main expert rules and 4^{th} additional rules which deal with exceptional cases. The later model developed by Howatson, Pullan et al. [10] uses a Monte Carlo method for growth of bifurcating systems in 3D space in response to host geometry.

Gillis and Lutchen [7] based on their 3D model have predicted images of ventilation distribution in asthmatic patients with potential clinical impacts. Other models e.g. [22] have been used to predict aerosol deposition in the human respiratory track which allows to plan inhaled aerosol therapy.

The models automatically generated by an algorithm [10, 11] consist of connected cylinders of different length and radius and have been constructed to study the structure-function relationship in the lung or to simulate flow and particle transport. Such models do not represent bronchial trees extracted from 3D CT of a chest (shortly called CT trees) whose branches reveal noised and geometrically deformed pipes. Therefore the models are insufficient for testing algorithms for quantitative analysis of CT trees.

On the other hand there is strong need for 3D models of CT trees. In last decade CT trees are increasingly being used in diagnosis and assessment of therapy of airway diseases. Early identification of patients with the use of computer aided CT trees analysis to assess the reconstruction can be an important step in the development of new treatments and new drugs that effectively prevent the development of adverse changes in the bronchial wall. Algorithms for 3D reconstruction of bronchial trees and measurement of local lumen of bronchial trees have to deal with deformations and noise in CT images e.g. [15, 18]. However, it is difficult to test the algorithms on CT images because correct values of local lumen are unknown. Therefore, Palagyi, Tschirren et al. have built simple phantom which consists of set of plastic tubes of constant diameter and smooth boundary surface [15]. Then they have made CT of them and have compared quantitative results given by computer algorithms with known, correct values [15]. Another strategy, presented in the paper, might be to automatically generate digital model, with the use of an algorithm, which take into account noise and geometric deformations in CT trees. Correct quantitative parameters of such models are known and can be compared with results of the algorithms being tested. Therefore, the main aim of the work is to construct an extension of the Kitaoka, Takaki et al. model [11], by adding noise and geometric deformations. The authors developed an algorithm which results in volumetric model of CT trees useful for testing algorithms for quantitative analysis of the trees. The model consists of two volumetric images: the first represents a tree map without noise and with geometric deformations, and the second contains a tree with noise and geometric deformations of branches. The proposed extension was developed based on the following assumptions:

Constant diameter of a branch: the analysis of a branch diameter allows us to obtain useful information about the tree being examined. Therefore it is very important to deform a branch in such a way that its diameter becomes constant. In this way correct value of a branch diameter is known which is necessary for testing of algorithms for quantitative analysis.

Noise and geometric deformations: data obtained from CT images very often contain different types of noise. Therefore, the presented model simulates these disruptions. The issue is extensively explained in section 3.3.

Efficiency in use: The generated bronchial tree model is planned to be used to test algorithms which measure local diameter of any branch of the generated tree. Therefore it is important to have fast access to correct local diameter for comparison. So, the value of local diameter is kept in each voxel- atom portion of a volume of generated tree. In general a voxel is defined by three coordinates and vector v of values represented by the voxel. In our case a voxel represents local diameter of a branch so v is a scalar.

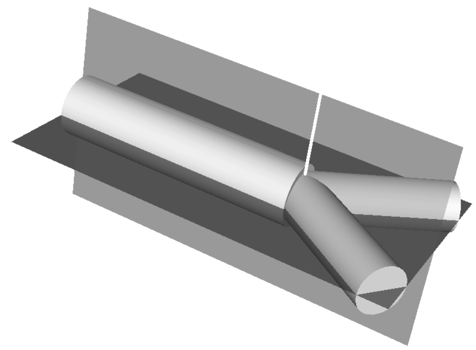


Fig. 2: Single bifurcation. Dark grey colour represents bifurcation plane, light grey colour represents volume division plane. Both planes are spread out to the borders of the parent region. The normal of the bifurcation plane has been marked with white color. This normal has been anchored in the bifurcation point

2 Basic model of bronchial trees

The chapter presents short description of an algorithm by Kitaoka, Takaki et al [11] called basic algorithm, for automatic 3D bronchial tree generation. Interested reader can find detailed explanation in [11]. The algorithm is based on three assumptions which make possible to realize regular and effective air transportation inside whole lungs.

The first assumption declares that each branch is a circular, rigid tube with a constant diameter.

The second assumption allows for correlation of the air transported through bronchi with its spatial position based on the following statement: the whole volume of air flowing through a branch is proportional to the volume of a lung that is supplied by this air. Furthermore, assume that for each bifurcation, children volume is proportional to the volume of its parent, which in turn means that the total air flow transported by the children is proportional to air flow transported by its parent.

The third assumption says that the final branches, which correspond to final bronchioles in the human respiratory tree, are homogeneously arranged within the organ.

The above assumptions allow to define described below expert rules which are fundamental for the construction of the basic algorithm. Successive execution of these rules allows to generate model of the bronchial tree. The rules and the assumptions presented above define branching geometry and are based on earlier morphometric studies and flow rate analysis in tubular living organs e.g. [9, 19]. Fig. 2 shows the basic parameters of a single bifurcation.

The algorithm requires input data that provide a set of parameters describing the root of the tree - trachea, and the surface (called boundary surface) that defines the space where the tree is generated.

Trachea: the most important parameters of trachea (root) are: diameter, length, and position in space. Moreover the root should be placed in the proper position relative to the abovementioned surface. Parameters of the trachea are chosen in accordance with the morphological

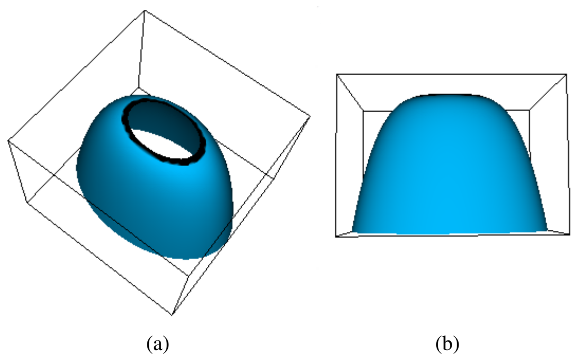


Fig. 3: The bounding surface which limits volume where the tree is generated, (a) view from the top, (b) side view

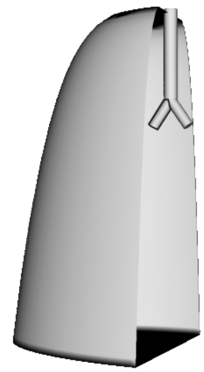


Fig. 4: Outer surface of a volume supplied by the left dauther of trachea

studies. Z coordinate of the end of the root, which defines area of first bifurcation, is set to 0.25 of the length of bounding surface in Z direction.

Volume: it is limited by the bounding surface, which can be defined in several ways. However, to simplify this work and following [11], we use a surface described by the equation 1 and shown in Fig. 3.

$$z = 2 \cdot 15^{-3}(x^2 + (1.5)^2)^2 \text{ for } 0 \leq z \leq 30 \qquad (1)$$

The following rules define the next steps, of an algorithm which successive execution until the end condition defined in rule 9, allows to generate the bronchial tree.

Rule 1: bifurcation is dichotomous, i.e., each parent is divided into two children.

Rule 2: longitudinal sections of a parent and his children are on the same plane, called the plane of bifurcation (see Fig. 2).

Rule 3: sum of the children flow is equal to the total flow of the parent: $d_0^n = d_1^n + d_2^n$, where d_0, d_1 and d_2 denote the diameter of the parent, first and second child respectively, the value of n is set to 2.8 based on statistical analysis of data from [16] and $d_1 \leq d_2$. Moreover, flow is described by the normalized equation.

Rule 4: volume, which is supplied by a parent is divided into two children volumes by the volume division plane. This plane is perpendicular to the plane of bifurcation and tends to the borders of the parent volume. Fig. 4 shows the volume determined under this rule.

Rule 5: the flow-dividing ratio $r \in (0; 0.5)$ is equal to the volume-dividing ratio, defined as the ratio of the volume of the smaller child to volume of its parent. The algorithm for r calculation is presented in [11].

Rule 6: diameter of two children and the bifurcation angle between them are defined as a function of r and parent diameter [11].

Rule 7: length of each branch is three times its diameter.

Rule 8: continuing to generate new branches in a given direction causes the children become new parents and their bifurcation plane is perpendicular to the bifurcation plane of their parent.

Rule 9: The process of generating new branches in a given direction continues until the flow rate for a new branch is less than minimal a-priori defined flow rate or when the branch goes beyond the region.

First two rules are based on observation of real lungs. The third rule is based on the modeling and analysis of inspiratory flow in bronchial trees. The fourth rule corresponds to the second assumption presented in this section. The sixth rule express the optimal relationships between flow rate, diameter and branching angles presented in earlier papers [9, 19]. Rule seventh is a result of examination of data presented in [16] which leaded to the

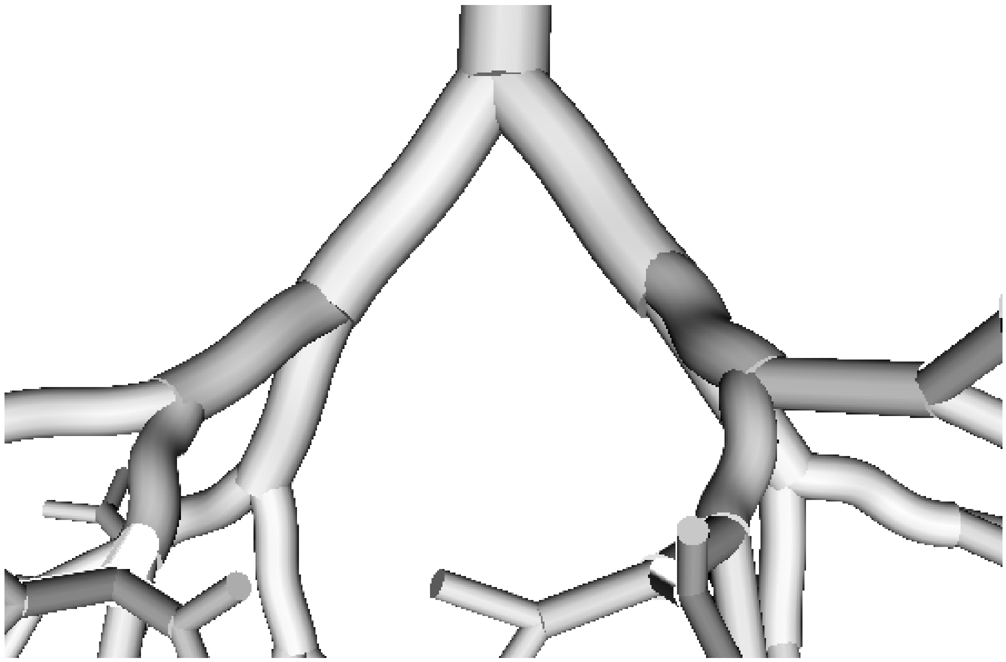


Fig. 5: Part of the model with bended branches indicated by arrows

conclusion that the distribution of the length-to-diameter ratio is Gaussian-like distribution with a mean of 2.8 and standard deviation of 1.0.

The additional four rules work in exceptional cases and are presented in details in [11].

2.1 Implementation issues of basic algorithm

In this section the authors describe the most important issues of implementation of the basic algorithm using VTK library [20] (Visualization ToolKit). VTK was chosen because it is dedicated for 3D image processing and visualisation, has large amount of classes which realize different 3D visualisation techniques and volumetric image processing algorithms, is portable, well documented and free. The clear structure of classes makes easy to find appropriate class to particular task of visualization or image processing.

The algorithm starts by determining the bounding space which defines the volume in which the bronchial tree is generated. Then it generates the root of the tree - trachea. The root and all branches are represented by cylinders. The use of a cylinder allows to obtain a uniform diameter of each branch, which was one of the crucial objectives of the implementation. However, there is a problem with cylindrical representation of branches. The connection of two cylinders of different radiuses generates unexpected sharp edges which do not occur in CT trees. The problem can be solved by the use of smoothing filter (see section 3.3), which smoothes the input object while preserving its topology.

Another important issue connected with geometrical transformations of branches can be simply solved by the use of *vtkTransform* class from VTK library. The class provides functions to define hierarchy of a transformation in parent branch local coordinate system, which facilitates the geometric transformation of its children. This hierarchy allows to manipulate of the parent without manually updating the coordinates of its children. Defining a hierarchy of transformation allows for significant simplification of the process of generating new branches. Moreover *vtkTransform* class has a method *TransformNormal* which in the case returns a normal vector of the transformed branch. The normal is used for determination of the branch division plane.

3 Extended algorithm of bronchial tree modeling

This chapter presents algorithm for bronchial tree modeling which is an extension of the basic algorithm. First, the extended algorithm generates basic model with the use of basic algorithm presented in the previous section. Then the branches generated in the first step are bent to obtain more "realistic" model. It returns triangulation of outer surface of the tree. Then, on the third step, the triangulated representation of the tree is transformed to voxel space. The fourth - last step consists in adding noise and smoothing to obtain a model which is more similar to real CT trees.

3.1 Geometric deformations of surface model

In the section the authors present branch bending procedure of generated bronchial tree. The procedure makes the generated model more similar to real segmented branches from CT images. The bending procedure generates a branch in two steps. First it draws a spiral with especially selected parameters and then the spiral is an input to the *vtkTubeFilter* class which is used to draw a "pipe" along the generated spiral. Drawing the spiral is realized according to equation 2 were f is a randomly selected function, from the following trigonometric functions: $\sin$, $-\sin$, $\cos$, $-\cos$.

$$
\begin{aligned}
z_k &= r \cdot f\left(2\pi \cdot x \cdot \frac{k}{n-1}\right), \\
x_k &= r \cdot f\left(2\pi \cdot x \cdot \frac{k}{n-1}\right), \\
y_k &= h \cdot \frac{k}{n}
\end{aligned} \tag{2}
$$

where: x - Rotation ratio. Specifies the number of twists of spiral. The value of this coefficient is selected depending on the degree of deformation of the branch. In our work x has been set to the value 1, r - Radius of rotation. In our work experimentally set to a branch radius divided by 3.5, n - number of voxels forming the spiral, k - index of a voxel in the spiral, h - length of the spiral in points.

An example of the effect of bending procedure is shown in Fig. 5. Moreover it is worth to emphasize that experiments carried out have shown that the bending procedure should be applied only to several first levels of the generated tree.

3.2 Transformations of the surface model to volumetric model

The bending procedure described in the previous section generates bronchial tree in the form of triangulation of its outer surface. The procedure works in continuous space because geometrical deformations are easy to perform in the space. The procedure of noise generation for a model of bronchial tree works in volumetric space (voxel space). Therefore after application of bending procedure the model is transformed to voxel representation, were image consists of voxels.

Conversion of the surface model into the volumetric model can be implemented in several ways. In the project only methods which fill inside of the converted object can be applied. Such methods can be divided into two groups, depending on whether the object can be represented as a set of other (usually simpler) objects, or should it be considered as a whole. If we want to consider the model as an

Fig. 6: Errors arising after conversion of a tree by a method from the class vtkPolDatToImageStencil

indivisible object we must to know about several issues. The most efficient conversion method available in VTK is based on *classvtkPolyDataToImageStencil*. The class performs conversion by searching for object inside points via propagation of rays along X axis, for each YZ slice of a 3D image. The propagated ray fills by voxels the space contained between each odd object point, met by the ray, and the next object point met by the ray. Therefore, in the case when the propagated ray touches boundary of an object having a "cavity", then it fills voxels which should be outside of the object. Exemplary errors that occur during this type of conversion are shown in the Fig. 6.

Satisfactory results can be obtained by conversion of the consecutive branches separately using *vtkPolyDataToImageStencil* class. This approach allows to assign to branch voxels proper diameter at the stage of conversion. For the sake of the property, this approach has been applied by the authors.

3.3 Distortions introduced into the volumetric model of bronchial tree

Distortions in volumetric space are generated in two steps. The first step consists in addition and subtraction of voxels to\from object with the use of modification of EDEN cell growing process [6] called later topological EDEN [3] or shortly topoEDEN. The algorithm preserves topology of a modified object what can be attained by modification only so called; simple voxels - voxels that addition or subtraction don't change the object topology. The algorithm for simple point detection in 3D has been presented and proved mathematically in [1].

In the case of bronchial trees the guarantee of topology preserving is very important because it ensures that during modifications of the tree any two branches does not merge. The topoEDEN procedure which erodes an input tree can be presented in several following steps:

topoEDEN(input: image I, number of iterations n, Output: image O)
$O = I$
take voxels from the border of O and put them into a list L.
for $i = 0; i < n; i++$ **do**
  choose randomly one voxel x from L.
  if x is simple **then**
    change its value in O to 0.
  end if
  delete x from L and update L.
end for
return O

The topoEDEN procedure can also dilate the input tree. In the case the procedure makes a list of border voxels of the background. Then it randomizes a voxel from the list and if it is simple assigns it to the tree and so on. There is also possible to make erosion and dilation alternately in topoEDEN procedure.

Example of topoEDEN way of working for 2D exemplary object is presented in Fig. 7. The input object (on the left) consists of two connected components and each of them has one cavity inside. The image on the right presents result of topoEden after several iterations (dark grey pixels). In the next iteration any of light grey pixels cannot be added to the input object because they change its topology (they are not simple pixels). They create new cavity or merge two components.

Exemplary results of topoEDEN procedure in 3D space

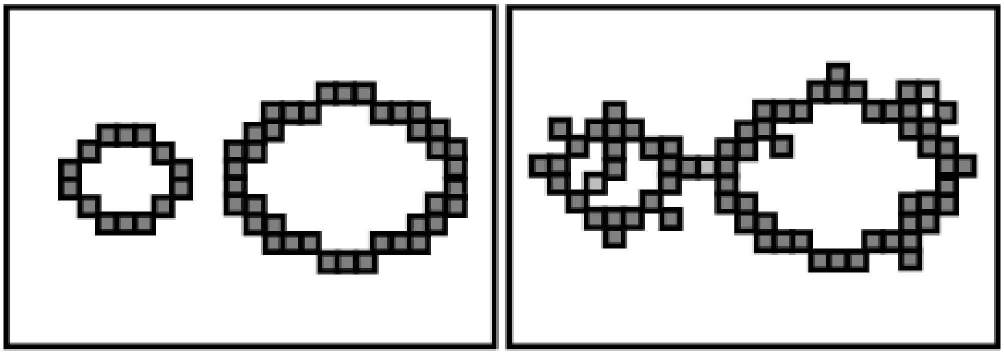


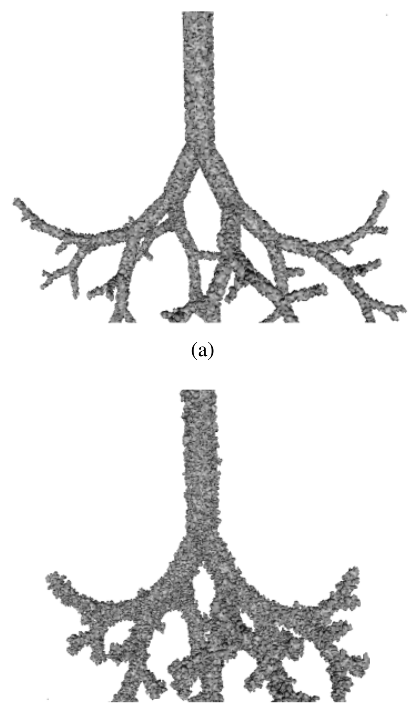
Fig. 7: Exemplary results of topoEden (version which adds points to an input object) for a hypotetical 2D object on the left. Pixels are represented with grey squares. Result of several iteration of topoEden on the right. Each of the light grey pixels cannot be added by topoEDEN in next iteration because they change topology of the input object

are shown in Fig. 8. Moreover, it is also possible to implement the procedure in such a way that it makes a list of the background and object border points, then if a randomly selected point from the list is simple it changes its assignment from tree voxel to the background or from background to the tree voxel.

Examples in Fig. 8 show that results of topoEDEN are not rewarding in terms of similarity to real segmented bronchial trees from CT images (see for example Fig. 10(a)). For that reason, at the next stage of processing the authors use iterative smoothing algorithm ASFT (*Alternate Sequential Filter controlled by Topology*) [5]. In this method smoothing is obtained by morphological open-close operation [17] with the use of sphere of variable radius. ASFT applied to result of topoEDEN gives more realistic view of a tree surface. Exemplary effect is shown in Fig. 9.

Finally, the extended model algorithm can be presented in the following several steps:

1. Generating of trachea and organ restricted area.
2. Generating of a tree branches based on basic algorithm rules.
3. Realization of individual tree branches flexion.
4. Conversion of the tree to the volumetric picture.
5. Iterative, alternate application of topoEDEN and ASFT operations.



(a)

(b)

Fig. 8: Exemplary results of topoEDEN when applied to a bronchial tree; (a) result for topoEDEN which erodes input object; (b) result for topoEDEN which dilates input object

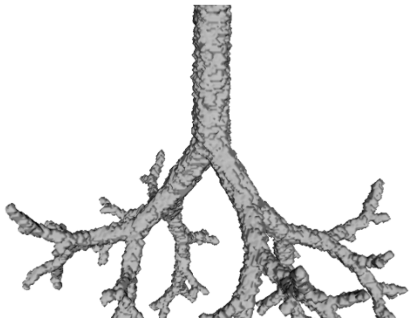


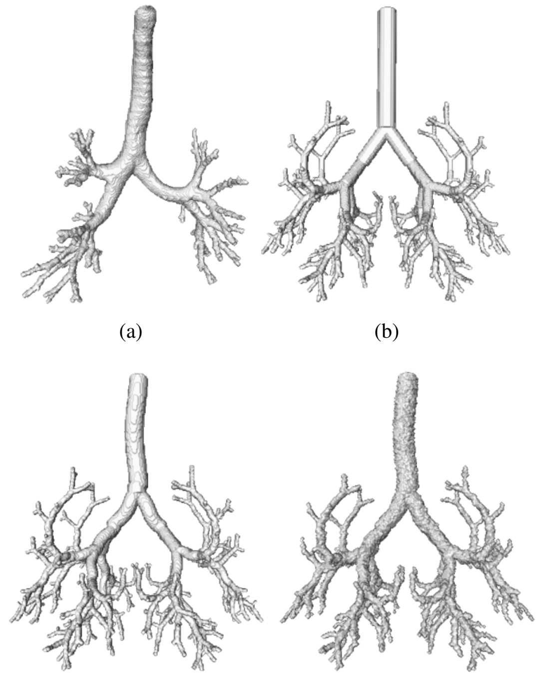
Fig. 9: The effect obtained by using topoEDEN (version which adds points to an input object) followed by ASFT

The algorithm has been implemented in C++ language with the use of VTK library.

4 Results

In this section the authors test topology of extended model experimentally and make comparison between basic model, extended model and CT trees. Moreover some results of testing of skeletonisation algorithm [14] on different bronchial tree models are presented. Fig. 10 presents example of basic model and extended model in order to show the reader the major differences between these two models. One can see that branches in extended model and CT tree are bent. Moreover the outer surface of CT tree is not as smooth as the outer surface of the basic model. Small local irregularities of the outer surface of the extended model make it more similar to CT tree than basic model.

The next experiment consisted in topology test of extended model. A bronchial tree is one connected component. It does not have any cavity and has as many tunnels as terminal branches. If any branch of a tree is clogged then its topology changes. Moreover if any two branches touch then a new tunnel is created and the topology changes. In order to test if extended model has the same topology as the corresponding basic model we have generated 10 trees for different parameters of bending procedure, topoEDEN and ASFT. Then for each tree



(a) (b) (c) (d)

Fig. 10: Bronchial trees generated with different models; (a) bronchial tree obtained by segmentation of tomographic image; (b) tree generated with the use of basic model; (c) basic model with bended branches; (d) extended model

we automatically calculated number of connected components and number of cavities. In order to check if there is no any hole inside the tree we have calculated number of connected components of the tree. There is no any hole in the tree if and only if the result is 1. In order to check if any two branches merge we applied tunnel closing algorithm [1] on a tree with the inside filled. Any two branches do not touch if and only if tunnel closing algorithm does not generate any tunnel closing patch. For all 10 generated trees we have obtained following results: one connected component, zero cavities, interior consisted for one component and hole closing algorithm has not generated any hole closing patch. So for 10 generated extended models all of them have the same topology as their corresponding basic models.

Fig. 11 shows results of skeletonisation algorithm ap-

Fig. 11: Skeletons of trees generated with different models; (a) basic model (b) extended model (c) CT tree

plied to basic model, extended model and CT tree. One can observe that the skeletonisation algorithm works almost perfectly on basic model while gives worse result on extended model and CT tree, i.e. significant number of unwanted branches are observed in Fig. 11(c).

The observation has been confirmed by quantitative analysis. We have generated one basic model, then based on the tree we have generated 10 extended models for different values of topoEDEN and ASFT. For an exemplary CT tree and the basic model the number of skeleton unexpected branches has been calculated. For 10 extended models the mean value of the number of unexpected branches has been calculated. All trees have 64 terminal branches which determines the same number of all branches. Results presented in Table 1 show that basic model is not suitable for testing skeletonisation algorithm as it is far less demanding than CT tree while extended models obtained mean value of unexpected branches similar to value of unexpected branches for CT tree. Therefore, extended model is useful for tests of skeletonisation algorithms.

The extended model algorithm have been prepared under guidance of pulmonologists and has been accepted by them for 3D print to obtain a phantom for testing of existing software for quantitative bronchial tree analysis based on CT of lung.

Tab. 1: Number of unexpected skeleton branches for different models and exemplary CT tree

	CT tree	Basic model	Basic model with bents	Extended models
Number of unexpected branches	56	6	5	57,3*

* the mean value of unexpected branches over 10 extended models

5 Final conclusions

The article presents conception of the new three dimensional spatial model of bronchial tree. The model is an extension of well-known approach to generate 3D representation of a bronchial tree [11] used for air flow calculations. The extension concerns geometric deformations and noise to make the model more similar to segmented bronchial trees from CT images. The authors have been shown experimentally that trees generated with extended model are more similar to CT trees. Moreover, the skeletonisation algorithm behaves similarly for extended models and CT trees, generating significant number of unwanted branches, while the same algorithm works almost perfectly on basic models. Future research will focus on the use of the extended model for testing algorithms to measure local lumen of bronchial trees. According to the authors knowledge the extended model is the first automatically generated by an algorithm model of CT trees which takes into account geometrical deformations of branches and noise.

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